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**ANALYTICAL APPROXIMATIONS**

**Volume 28**

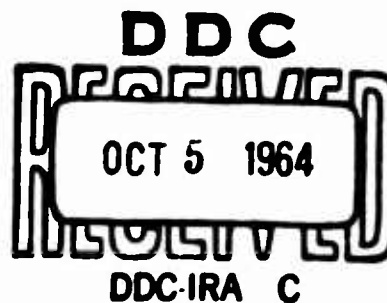
**Cecil Hastings, Jr.  
Elaine Hastings**

**P-1301 ✓**

**4 March 1958**

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### Analytical Approximation

Gaussian Error Integral: To better than .0005 over  $(-\infty, +\infty)$ ,

$$\begin{aligned} \mathfrak{I}(x) &= \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \\ &= \frac{x}{\sqrt{x^2 + (a_0 + a_2 x^2 + a_4 x^4)^{-4}}} \end{aligned}$$

wherein

$$a_0 = 1.0635$$

$$a_2 = .1535$$

$$a_4 = .0341$$

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### Analytical Approximation

Gaussian Error Integral: To better than .00004 over  $(-\infty, +\infty)$ ,

$$\begin{aligned} \mathcal{E}(x) &= \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \\ &= \frac{x}{\sqrt{x^2 + (a_0 + a_2 x^2 + \dots)^{-4}}} \end{aligned}$$

wherein

$$a_0 = 1.06214$$

$$a_2 = .16193$$

$$a_4 = .02431$$

$$a_6 = .00266$$

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### Analytical Approximation

Gaussian Error Integral: To better than .000006 over  $(-\infty, +\infty)$ ,

$$E(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

$$= \frac{x}{\sqrt{x^2 + (a_0 + a_2 x^2 + a_4 x^4 + \dots)^{-4}}}$$

wherein

$$\begin{aligned} a_0 &= 1.062273 \\ a_2 &= .160871 \\ a_4 &= .026161 \\ a_6 &= .001648 \\ a_8 &= .000158 \end{aligned}$$

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### Analytical Approximation

Gaussian Error Integral: To better than .00007 over  $(-\infty, +\infty)$ ,

$$\begin{aligned} \mathfrak{I}(x) &= \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \\ &\doteq \frac{x}{\sqrt{x^2 + (a_0 + a_2 x^2 + a_4 x^4)^{-8}}} \end{aligned}$$

wherein

$$a_0 = 1.03074$$

$$a_2 = .07761$$

$$a_4 = .01019$$

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### Analytical Approximation

Gaussian Error Integral: To better than .000006 over  $(-\infty, +\infty)$ ,

$$I(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

$$= \frac{x}{\sqrt{x^2 + (a_0 + a_2 x^2 + a_4 x^4 + a_6 x^6)^{-8}}}$$

wherein

$$a_0 = 1.030653$$

$$a_2 = .078127$$

$$a_4 = .009592$$

$$a_6 = .000157$$

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### Analytical Approximation

Gaussian Error Integral: To better than .000005 over  $(-\infty, +\infty)$ ,

$$\bar{I}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

$$= \frac{x}{\sqrt{x^2 + (a_0 + a_2 x^2 + a_4 x^4)^{-10}}}$$

wherein

$$a_0 = 1.024457$$

$$a_2 = .062087$$

$$a_4 = .007205$$

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